UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Level

THINKING SKILLS **9694/03**

Paper 3 Problem Analysis and Solution **For Examination from 2011**

SPECIMEN PAPER

1 hour and 30 minutes

Additional Materials: Answer Booklet/Paper

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Electronic calculators should be used.

Answer **all** the questions.
Start each question on a new answer sheet.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question.

This document consists of **9** printed pages and **1** blank page.

1 *Study the information below and answer the questions. Show your working.*

Alfonso and Benito send messages in Italian using the following cipher system:

- They always use the same table of 5 rows and 5 columns for the letters.
- They randomly place the numbers 0 to 9 in two columns to the left of the table of letters.
- They then place another random selection of the numbers 0 to 9 in two rows above the table of letters.
- Each character of a message is then encoded into a two-digit number:
  - the first digit is obtained by selecting one of the two numbers directly to the left of the character,
  - the second digit is obtained by selecting one of the two numbers directly above the character.

An example is shown below. (There are only 21 letters in the Italian alphabet.)

		1	7	5	2	3
		8	4	0	9	6
6	3	**A**	**B**	**C**	**D**	**E**
0	7	**F**	**G**	**H**	**I**	**L**
9	4	**M**	**N**	**O**	**P**	**Q**
2	8	**R**	**S**	**T**	**U**	**V**
5	1	**Z**	**.**	**,**	**-**	**(space)**

In this example, **Q** could be 93, but it could also be represented as 96, 43, or 46. The choice is the sender's.

(a) Using the example above, write down the word given by the following encoding:

61	03	06	33	77	88	45

[1]

(b) Using the example above, give **two** different ways to encode **UNO**. [2]

The next day, they agreed a different random placement of the numbers.
Alfonso sent the following message:

87	36	68	89	72	83	68	12	17	82	80	70	84

Shortly afterwards, Alfonso sent the same message again, because he realised that he had made a mistake. In fact, he had put the numbers intended for the top down the side, and the side ones on the top, but he *had* used the pairs in the right order. The re-sent message was as follows:

40	60	12	45	47	25	86	07	00	28	41	44	48

By comparing the two messages, we can see that where there was an 8 or a 7 as the first digit of a two-digit number, there should have been a 4 or a 2.

(c) Reconstruct all the **correct** pairs of numbers. Your answer should include the table below, completed fully. You do not need to worry about the order of the columns. [4]

Correct 1st digit (side)	4 or 2				
Correct 2nd digit (top)	8 or 7				

The appearance of 68 twice, in the 3rd and 7th positions of the message as originally sent, indicates that they are both the same character, and we can assume that it is a space.

(d) For four of the other characters in the message, we can be sure that each of them is either an **A**, a **G**, an **O** or a **U**. Which four positions are they in? [1]

(e) Given that the first letters of the three words in the message are **M**, **N** and **T** respectively, write out the decoded message. (You can use the table below to help if you wish, but be sure to include your working with your answer.) [2]

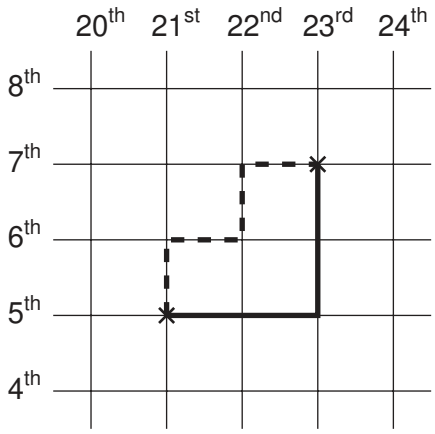
		A	**B**	**C**	**D**	**E**
		F	**G**	**H**	**I**	**L**
		M	**N**	**O**	**P**	**Q**
		R	**S**	**T**	**U**	**V**
		Z	**.**	**,**	**-**	**(space)**

2 *Study the information below and answer the questions. Show your working.*

In the world of taxi drivers who work in a city with a regular grid, a *straight line* is defined as a shortest path between two points. Sometimes there can be more than one, as demonstrated in the example below.

In this case, between the corner of 21st (Avenue) and 5th (Street) and 23rd and 7th, the minimum path length (distance) is 4.

We write these as (21,5) and (23,7). Two possible paths are:



and there are more such paths.

(a) The following journeys all have a shortest path of 5. What is the total number of paths of length 5 in each case?

(i) From (3,4) to (8,4). [1]

(ii) From (4,5) to (8,4). [1]

(iii) From (6,7) to (8,4). [1]

Automatic equipment for storage has a similar way to measure distance when items are moved from one cube to another in three dimensions. In this case we need to use 3 numbers (coordinates) to specify where an item is. Of course, the coordinates can only be whole numbers.

In this world of automatic storage, a *sphere* is a set of points which are all the same distance from the centre.

(b) List all the points on the *sphere* of distance 1 from (3,4,5). [2]

(c) How many shortest paths are there from (4,3,3) to (3,4,5)? [3]

There's no need to stop at just using three dimensions.

(d) A four-dimensional hyper-taxi driver needs to get from (3,1,3,1) to (2,2,2,2). How many shortest paths in hyper-taxi space are there? [2]

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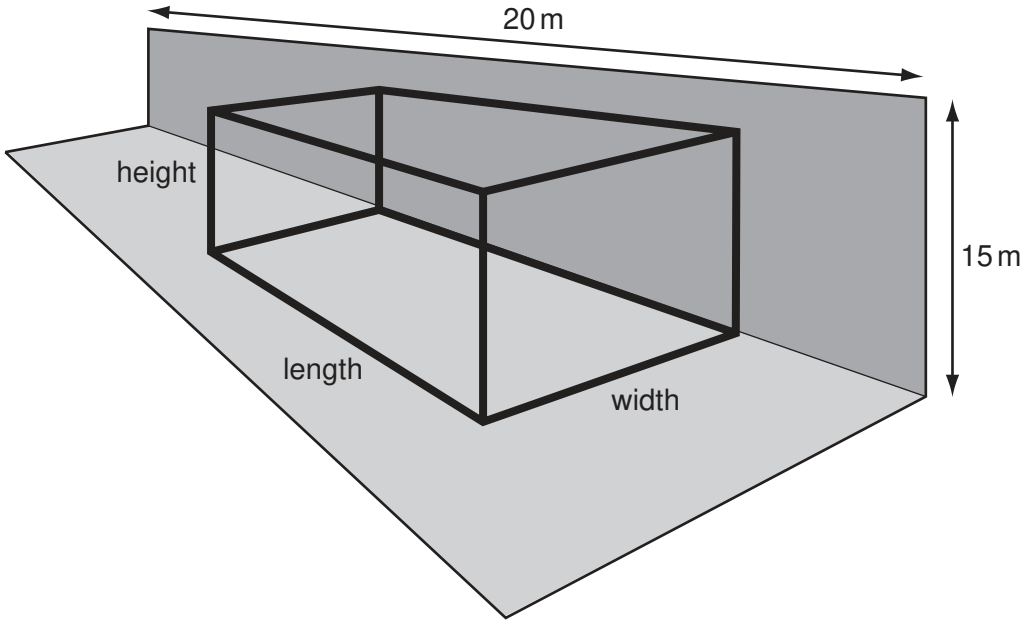
[Question 3 is printed on the next page]

3 *Study the information below and answer the questions that follow. Show your working.*

A cage is to be built to house some exotic birds. It is to be built against a wall, as shown in the diagram, and thus needs 12 struts altogether and netting on four sides (i.e. the three vertical faces and the roof). The structure must be in the shape of a cuboid and have dimensions which are whole numbers of metres.



Physical constraints

Throughout this question all your answers must abide by the following three constraints.

- The wall is **15 metres high** and **20 metres long**.
- There is **600 m^2 of netting** available.
- The **smallest strut which can be used for the framework is 9 metres long**, so none of the dimensions of the cage can be less than this.

Desirable outcome

For part **(d)** of this question you will also need to consider the following additional outcome.

- For animal welfare reasons the cage should have a volume of **1900 m^3** or more.

The final design of the cage will fulfill this desirable outcome, but the process of designing the cage may involve considering dimensions which do not do this.

An example of the dimensions of a cage which could be made from the netting, **but which does not fit the animal welfare consideration**, is given below. Note that the dimension referred to as 'length' is that which is parallel to the wall.

Example

Length: 11 m
Width: 12 m
Height: 13 m

Area of netting required: roof = 12 × 11 = 132 m^2
front = 13 × 11 = 143 m^2
sides = 2 × (13 × 12) = 312 m^2
TOTAL = 587 m^2 (which is within specified limit of netting area)

Volume = 13 × 12 × 11 = 1716 m^3 (which does not satisfy the specified limit of cage volume)

(a) If a cage with dimensions 10 m, 11 m and 15 m was to be constructed, what is the **smallest total area** of netting that would be needed? [2]

(b) Consider possible cages which are of the same height as the wall. What is the **maximum length** that the cage can be? Justify your answer. [3]

(c) What is the **largest volume** cage that can be built with a height of 14 metres? Justify your answer. [3]

(d) Show that it is possible to fulfil the desired animal welfare outcome of a minimum volume of 1900 m^3. Give the length, width and height of the cage which does this. [3]

(e) It is decided that the wall is bad for the birds' welfare, and is to be demolished. Plans are drawn up for the new cage [which must have 5 netted sides, now the wall has gone].

Four 10 metre struts are found, and it is decided that these should be used for the cage [i.e. at least one of the dimensions should be 10 metres]. The other struts are subject to the same restrictions as before.

What is the **minimum extra amount of netting** needed if the cage is still to fulfil the 1900 m^3 requirement? [4]

4 *Study the information below and answer the questions that follow. Show your working.*

The Island of Ferrin is linked to the mainland by a causeway (road). The island becomes cut off from the mainland twice a day as the incoming tide covers the road. It is possible to make accurate predictions of when it is possible to cross the causeway. Crossing times are published well in advance for the benefit of residents and tourists. The table below shows the crossing times for next July. For example, on 1 July the causeway is open from 10:15 to 16:35 and then from 22:05 to 05:00 the following day.

	CAUSEWAY OPEN		**CAUSEWAY OPEN**	
July	**From**	**To**	**From**	**To**
1	10:15	16:35	22:05	05:00
2	10:35	17:40	22:30	06:15
3	10:55	18:55	23:05	07:50
4	11:45	20:25		
5	00:10	09:20	13:05	21:45
6	01:40	10:35	14:45	22:50
7	03:15	11:30	16:10	23:35
8	04:40	12:10	17:15	00:15
9	05:45	12:45	18:05	00:45
10	06:35	13:15	18:40	01:15
11	07:10	13:40	19:10	01:50
12	07:40	14:05	19:40	02:25
13	08:05	14:40	20:05	03:00
14	08:25	15:15	20:30	03:40
15	08:45	15:55	20:55	04:25
16	09:05	16:35	21:15	05:10
17	09:25	17:25	21:40	06:05
18	09:55	18:20	22:10	07:10
19	10:35	19:25	22:55	08:20
20	11:40	20:35		
21	00:10	09:30	13:15	21:45
22	01:40	10:30	14:55	22:35
23	03:20	11:15	16:25	23:15
24	04:45	11:50	17:40	23:50
25	06:05	12:20	18:45	00:20
26	07:10	12:55	19:40	01:00
27	08:10	13:35	20:25	01:45
28	07:55	13:25	20:00	01:40
29	08:30	14:20	20:25	02:40
30	08:55	15:15	20:50	03:50
31	09:20	16:25	21:20	05:10

Bus times

The mainland end of the causeway is in the village of Linkham. A local bus service runs from Barwell to Linkham. Upon arrival at Linkham the bus continues on to Ferrin. The causeway must be open for the bus's outbound time and the inbound time at Linkham. For instance, for the first bus of the day to make the crossing, the causeway must be open for the whole of the time between 09:26 and 10:19 (see table below). Here is the Bus Timetable (which operates daily):

Outbound	Barwell (Station)	09:05	10:10	12:10	14:15	15:05	16:15	18:10
	Barwell (High Street)	09:10	10:15	12:15	14:20	15:10	16:20	18:15
	Hayston	09:24	10:29	12:29	14:34	15:24	16:34	18:29
	Linkham	09:26	10:31	12:31	14:36	15:26	16:36	18:31
	Ferrin (Castle) – arrive	09:40	10:45	12:45	14:50	15:40	16:50	18:45
Inbound	Ferrin (Castle) – depart	10:05	11:05	13:05	15:05	15:45	17:05	18:45
	Linkham	10:19	11:19	13:19	15:19	15:59	17:19	18:59
	Hayston	10:21	11:21	13:21	15:21	16:01	17:21	19:01
	Barwell (High Street)	10:35	11:35	13:35	15:35	16:15	17:35	19:15
	Barwell (Station)	10:40	11:40	13:40	15:40	16:20	17:40	19:20

Castle opening times

Ferrin Castle is the Island's main tourist attraction. It opens each day at 09:00 or 30 minutes after the causeway opens (whichever is the later) and closes at 18:00 or 30 minutes before the causeway closes (whichever is the earlier). However, on days when it is not possible to be open for a continuous period of at least three hours it remains closed.

(a) State the arrival times at the Castle bus stop of all the buses that will continue from Linkham to Ferrin on 24 July. (The bus makes the journey whether or not the Castle is open). [3]

(b) At what time will the Castle open, and at what time will it close:

(i) On 17 July? [2]

(ii) On 20 July? [2]

(c) The first date in July when the Castle will be closed all day is 6 July.
On what date will the Castle next open? Show clearly how you reach your conclusion. [2]

(d) Barry and Clare will be staying in Barwell from 2 July to 12 July. During their holiday they would like to take the bus to Ferrin. However, they will only do so if they will be able to spend at least five hours exploring the Island, including a visit to the Castle.

On which date(s) would this be possible?

Show all your working, and for each date that is possible state the earliest arrival time at, and the latest departure time from, the Castle bus stop. [6]

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